B.Sc. (Honours) Examination, 2020
Semester-IV
Statistics
Course: CC-7
(Mathematical Analysis [Theory and Tutorial])
Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin
Notations have their usual meanings

Group – A (Answer any ten questions) $10 \times 1 = 10$

1. Answer the following questions with proper justification.

   (a) Give an example of a set which is not ordered field.

   (b) How do you represent $\sqrt{8}$ on directed line?

   (c) Write down the definition of equivalent sets and give an example.

   (d) Find $Sup(S)$ and $Inf(S)$ of $S = \{|x| : x^2 < 1, x \in R\}$.

   (e) Discuss if $\{\tan \frac{10\pi}{n}\}$ is a sequence or not.

   (f) Justify whether every convergent sequence is bounded or not.

   (g) Suppose $\sum u_n$ is an infinite series with $\lim\limits_{n\to\infty} u_n = 0$. What can you conclude about the series convergence?

   (h) If $\sum u_n$ be a convergent series of positive real numbers then what can you say about the convergence of series $\sum u_n^4$.

   (i) How to represent a step function by using indicator function?

   (j) Is $f(x) = [x]$, $x \in \boldsymbol{R}$ a bijective function?

   (k) Find $\lim\limits_{x\to 0} Sgn(x)$.

   (l) What do you mean by uniform continuity?

   (m) How you can derive Lagrange's mean value theorem from Cauchy's mean value theorem?

   (n) What is the relation between ∇ and Δ operator?

Group – B (Answer any five questions) $5 \times 6 = 30$

2. (a) Show that $2.7^n + 3.5^n - 5$ is divisible by 12.
   (b) Show that the set of rational numbers $\boldsymbol{Q}$ is not order complete. 3+3

3. (a) Show that $\{n^{\frac{1}{n+1}}\}$ converges to 1.
   (b) Find $\lim\limits_{n\to\infty}\left[\frac{1^2}{n^3} + \frac{2^2}{n^3} + \cdots + \frac{n^2}{n^3}\right]$. 3+3

4. (a) State and prove Cauchy's first limit theorem on sequence.
(b) Show that the sequence $\{x_n\}$ where $x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$ is convergent. 4+2

5. (a) Discuss the statement: Every bounded sequence is a Cauchy sequence.
(b) Suppose $x_n = (1 - \frac{1}{n^2})\sin(\frac{n\pi}{4})$, then find two subsequences of $\{x_n\}$ one of which converges to the $\overline{\lim}\, x_n$ and other converges to the $\underline{\lim}\, x_n$. 3+3

6. (a) Comments on the series $\sum(\sqrt[3]{n^4+1} - \sqrt[3]{n^4-1})$.
(b) Discuss the convergence of the series $\frac{2+k}{3+k} + \frac{2^2+k}{3^2+k} + \frac{2^3+k}{3^3+k} + \cdots$. 3+3

7. (a) If $\sum u_n$ be a convergent series of positive real numbers, then show that $\sum \frac{u_n}{1+u_n}$ is also convergent.
(b) State the alternative form of Gauss test. Discuss the convergence of the series $1 + \frac{\alpha}{1!} + \frac{\alpha(\alpha+1)}{2!} + \frac{\alpha(\alpha+1)(\alpha+2)}{3!} + \cdots$. 2+4

8. (a) State and prove Leibnitz's test.
(b) Discuss the conditional convergence of the series $\sum \frac{(-1)^{n-1}}{(n+1)^2 \log(n+1)}$. 3+3

9. (a) Show that $\lim\limits_{x\to 0} \sin^3\left(\frac{1}{x}\right)$ does not exists.
(b) Let $f : (0, \infty) \to \boldsymbol{R}$ be defined by $f(x) = x(e^{\frac{1}{x^3}} - 1 + \frac{1}{x^3})$. Then find out correct statement(s). (i) $\lim\limits_{x\to\infty} f(x)$ exists. (ii) $\lim\limits_{x\to\infty} xf(x)$ exists. (iii) $\lim\limits_{x\to\infty} x^2 f(x)$ exists. (iv) There exists $m > 0$ such that $\lim\limits_{x\to\infty} x^m f(x)$ does not exists. 3+3

Group – C (Answer any two questions) $2 \times 10 = 20$

10. (a) If f and g be two functions which are continuous at c, then show that $f(x)g(x)$ is continuous at c.
(b) Let $f(x) = x|x| + |x-1|,\ x \in \boldsymbol{R}$. Then which of the following statements is true? (i) f is not differentiable at $x = 0, 1$. (ii) f is differentiable at $x = 0$ but not differentiable at $x = 1$. (iii) f is not differentiable at $x = 0$ but differentiable at $x = 1$. (iv) f is differentiable at $x = 0, 1$.
(c) If $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \ldots + \frac{a_n}{n+1} = 0$, where $a_i \in R$, show that the equation $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n = 0$ has at least one real root between $(0, 1)$. 4+3+3

11. (a) Let a function $f : [0, 1] \to \boldsymbol{R}$ be continuous on $[0, 1]$ and differentiable on $(0, 1)$. If $f(0) = 1$ and $[f(1)]^3 + 2f(1) = 5$, then prove that $\exists$ a $c \in (0, 1)$ such that $f'(c) = \frac{2}{2+3[f(c)]^2}$.
(b) Using Taylor's theorem or otherwise show that $x - \frac{x^3}{3!} < \sin(x) < x - \frac{x^3}{3!} + \frac{x^5}{5!}$, for $x > 0$.
(c) Obtain the power series expansion of $\cos(x)$. 3+3+4

12. (a) Let a function $y = f(x)$ have the values $y_0, y_1, \ldots y_n$ corresponding to the values of the argument $x_0, x_0 + h, \ldots, x_0 + nh$, then for any non-negative integer m, show that $\Delta^m y_r = \sum\limits_{i=0}^{m} (-1)^{m-i} \binom{m}{i} y_{r+i}$.
(b) State and prove Gauss's Forward Interpolation Formula.
(c) Find the solution of the difference equation $x(n+1) - 3^n x(n) = 0$. 3+4+3